

# Math Wrangle Practice 2

## Marin Math Circle

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1. A tennis player computes her win ratio by dividing the number of matches she has won by the total number of matches she has played. At the start of a weekend, her win ratio is exactly 0.500. During the weekend, she plays four matches, winning three and losing one. At the end of the weekend, her win ratio is greater than 0.503. What is the largest number of matches she could have won before the weekend began?
2. For how many values of  $k$  is  $12^{12}$  the least common multiple of the positive integers  $6^6$ ,  $8^8$ , and  $k$ ?
3. Let  $S$  be a finite, nonempty set of real numbers such that the distance between any two distinct points in  $S$  is an element of  $S$ . In other words,  $|x - y|$  is in  $S$  whenever  $x \neq y$  and  $x$  and  $y$  are both in  $S$ . Prove that  $S = \{a, 2a, 3a, 4a, \dots\}$  for some number  $a$ . (BAMO 2011)
4. Suppose that  $n$  squares of an infinite square grid are colored grey, and the rest are colored white. At each step, a new grid of squares is obtained based on the previous one, as follows. For each location in the grid, examine that square, the square immediately above, and the square immediately to the right. If there are two or three grey squares among these three, then in the next grid, color that location grey; otherwise, color it white. Prove that after at most  $n$  steps all the squares in the grid will be white. (BAMO 2006)
5. A grasshopper is sitting in the corner of a square room with side 2 meters. Each jump covers exactly 2 meters. He starts to jump. What points in the room can he reach?
6. Is it true that a square can always be cut into any number of squares (not necessarily equal), beginning with 6?
7. Is it possible to find 10 different numbers, the product of any two of which will be divisible by the sum of all 10 numbers?
8. Let  $P_0(x) = x^3 + 313x^2 - 77x - 8$ . For integers  $n \geq 1$ , define  $P_n(x) = P_{n-1}(x - n)$ . What is the coefficient of  $x$  in  $P_{20}(x)$ ?