MATH AUCTION

October 14, 2012

Rules:

1) We divide into teams and work for a fixed amount of time to solve the problems below.

2) Each team is given $500 to start.

3) The best solution to a problem is worth $200.

4) The problems are put up for auction in the order given. The team with the highest bid is allowed to present its solution.

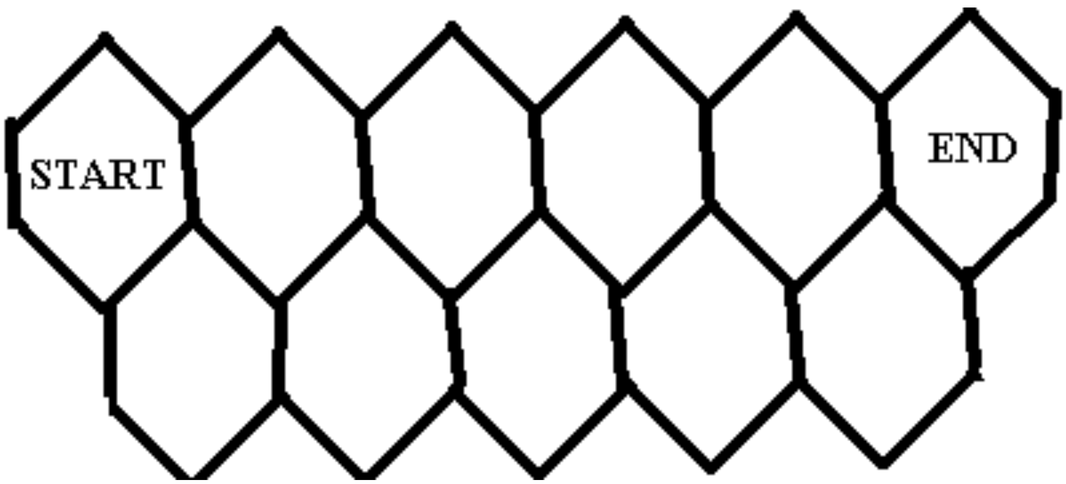
5) The problem is then put up for bid again (and again), but each time the solution must be better than the previous solution.

6) When no other team wants to buy the problem, the team with the best solution collects the value of the problem. Every team that "bought" the problem pays for its bid, even if it did not have the winning solution.

Problems:

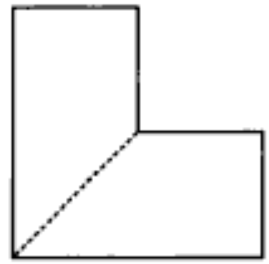
1. A tower of 80 coins is placed at the central square of a 1 × 1001 board. During a turn, one can lift the top k coins from any tower (you can lift any number of coins, even all coins), and place them onto a square k fields to the right or to the left. If this square contains some coins already, then the relocated coins are placed on top. The goal is to relocate all coins from the original position to the square immediately right from it. Do this in as few turns as possible.

2. A honeycomb is drawn below. In how many different ways can you walk from "start" to "end", only ever moving one cell to the right, or cell diagonally down, or one cell diagonally up?

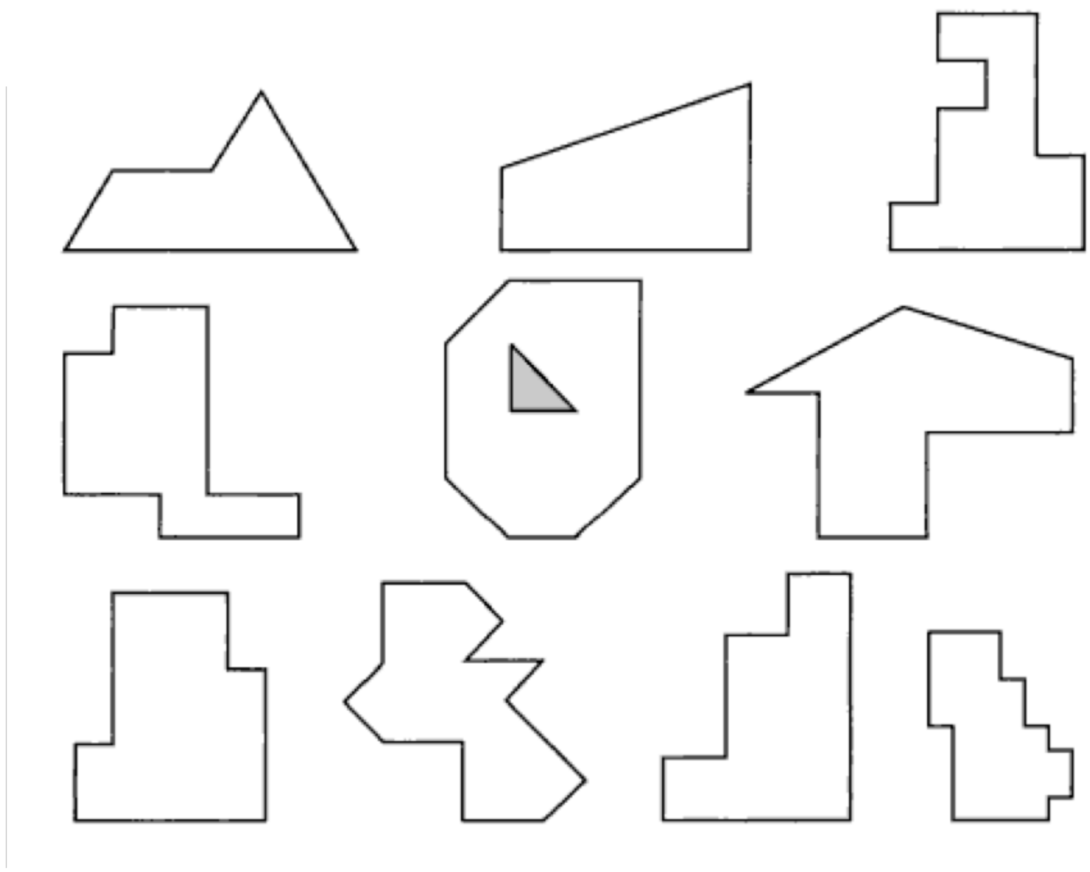


3. There are 100 computers. At least 51 are good, the rest - at most 49 - are broken. You can ask computer A if computer B is ok or not. If A is good, it'll give the correct answer, but if it's bad, it can give any answer. Your task is to locate a good computer as efficiently as possible (specifically, find a protocol minimizing the number of questions in the worst case).

4. This L shape can be divided into two congruent halves. (Mirror images are considered congruent.)



Divide as many of these shapes as you can into two congruent halves.



5) A military base has a number of identical hoverplanes. Each hoverplane can carry enough fuel to fly exactly halfway around the planet. Hoverplanes do not use any fuel while hovering stationary in the air, and hoverplanes can transfer any amount of fuel between each other while in the air. What is the minimum number of planes are that are needed so that one plane is able to get all the way around the planet and all assisting planes return safely to base.

Thank you to the Northwest Academy of Sciences Math Circle for problem 1, the Math Circle in Boston for problem 2, Alon Amit for problem 3, *Solve This* by James Tanton for problem 4, and the Bay School for problem 5.