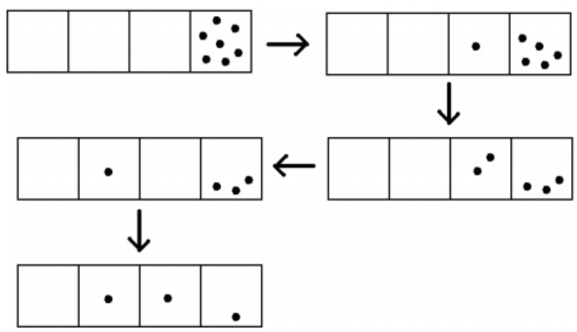
Exploding Dots!

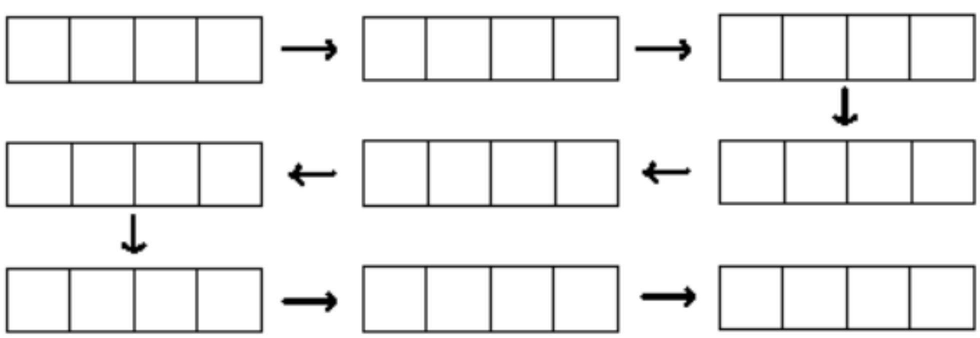
We will use Exploding Dots, a.k.a Dots and Boxes, to explore place value, changing bases, and operations on numbers. It starts like this:

THE 1←2 RULE:
Whenever there are two dots in any one box they "explode," disappear and become one dot in the next box to their left

Here are nine dots placed in the rightmost box. Can you see how these dots are exploded with the 1 ← 2 rule?



We encode 7 as 111 in the 1 ← 2 system.
Try encoding ten in the 1 ← 2 system

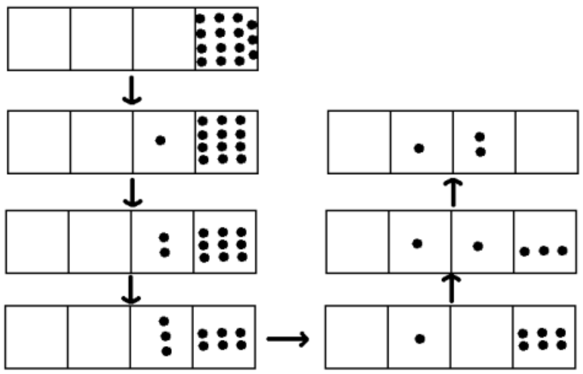


What is the code for 13 in the 1 ← 2 system?
What is the code for 6 in the 1 ← 2 system?
What is your age coded in the 1 ← 2 system?
What number is coded as 101 in 1 ← 2 system?

Here is another rule:

THE 1 ← 3 RULE:
Whenever there are three dots in any one box they "explode," disappear and become one dot in the next box to their left

Let's begin with 15 dots. Follow how we explode these dots to encode 15 in the 1 ← 3 system
What is the code for 13 in the 1 ← 3 system?



What is the code for 6 in the 1 ← 3 system?
What is your age coded in the 1 ← 3 system?
What number is coded as 101 in 1 ← 3 system?

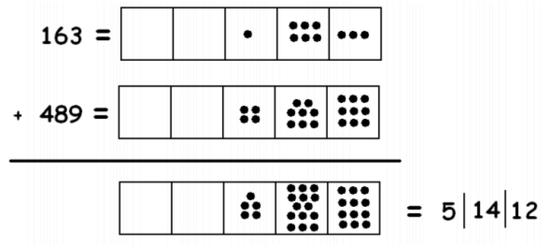
Compare these two systems:

What did you notice?

	Encoding	
Number	1 ← 2 system	1 ← 3 system
15		
9		
13		
16		
	101	
		101

How many dots does it take in the rightmost box to make 10 in a 1 ← 2 system?
How many dots does it take in the rightmost box to make 100 in a 1 ← 2 system?
How many dots does it take in the rightmost box to make 10 in a 1 ← 3 system?
How many dots does it take to in the rightmost box make 100 in a 1 ← 3 system?
What system do we normally work in?

It gets better! Let's add in a 1 ← 10 system.



What is going on here?

$$\begin{array}{r} 163 \\ +\ 489 \\ \hline 5\ \ 14\ \ 12 \end{array}$$

Try to add several other 3 or 4 digit numbers together using this method.

$$\begin{array}{r} 148 \\ +\ 323 \\ \hline = \end{array} \qquad \begin{array}{r} 567 \\ +\ 271 \\ \hline = \end{array} \qquad \begin{array}{r} 377 \\ +\ 188 \\ \hline = \end{array} \qquad \begin{array}{r} 582 \\ +\ 714 \\ \hline = \end{array}$$

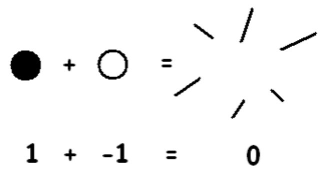
$$\begin{array}{r} 310462872 \\ +\ 389107123 \\ \hline = \end{array} \qquad \begin{array}{r} 87263716381 \\ +\ 18778274824 \\ \hline = \end{array}$$

Challenge

Can you extend this to multiply in a 1 ← 10 system?

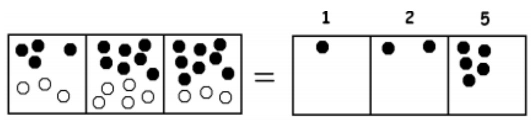
The Anti-Dot

To do subtraction we will add another character, the Anti-dot! The Anti-dot annihilates a dot it is paired with.



Now we can do subtraction. Let's subtract 353 from 478.

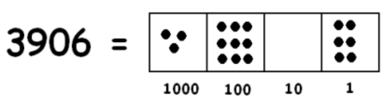
$$\begin{array}{r} 478 \\ -\ 353 \\ \hline =\ 125 \end{array}$$



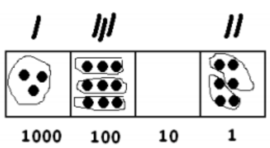
Can you show 423 – 254 using this method? (*hint: you may have to implode!*)

Now lets look at division! We will divide 3906 by 3.

Here is 3906 and 3



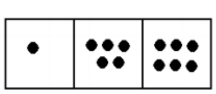
We circle the groups of 3 in 3906.



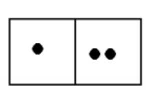
The answer is 1302!

Let's try 156 divided by 12. Can you show the groups of 12 in 156?

156



12

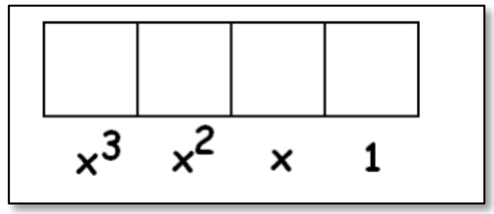


ADVENTURE 2: Anu refuses to tell anyone if she is working in a $1 \leftarrow 10$ system, or a $1 \leftarrow 5$ system, or any other system. She makes everyone call it an $1 \leftarrow x$ system but won't tell a soul what number she has in mind for x.

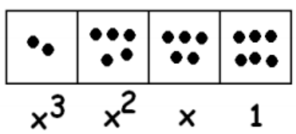
We know that boxes in a $1 \leftarrow 10$ have values that are powers of ten: 1, 10, 100, 1000, 10000, …

And boxes in a $1 \leftarrow 5$ system powers of five: 1, 5, 25, 125, 625, …

So Anu's system, whatever it is, must be powers of x:

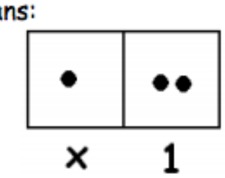


When Anu writes 2556 she must mean:



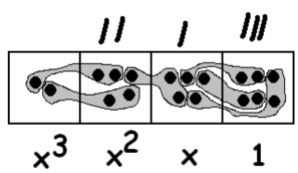
$$2x^3 + 5x^2 + 5x + 6$$

and when she writes 12 she means:



$$x + 2$$

Anu decides to compute $2556 \div 12$. She obtains:



$$\left(2x^3 + 5x^2 + 5x + 6\right) \div \left(x + 2\right) = 2x^2 + x + 3$$

Whoa!

a) Use Anu's method to find $\left(3x^2 + 7x + 2\right) \div \left(x + 2\right)$

b) Use Anu's method to find $\left(2x^4 + 3x^3 + 5x^2 + 4x + 1\right) \div \left(2x + 1\right)$

c) Use Anu's method to find $\left(x^4 + 3x^3 + 6x^2 + 5x + 3\right) \div \left(x^2 + x + 1\right)$