Prime numbers and irreducible polynomials

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1. Is the polynomial X^2+4X+3 irreducible? What about X^2+3X+4?
2. Show that $X^3-5X+14$ is irreducible. What about $X^3-51X+14$?
3. Show that X^4+1 is irreducible, but X^4+4 is reducible.
4. Show that the polynomial
$$X^5+2X^3+2X+4$$
is irreducible.
5. Show that the polynomial
$$X^5+6X^4+6X^3+24X+72$$
is irreducible.
6. Show that if p is a prime number and q is not divisible by p, then X^n-pq is irreducible.
7. Show that the polynomial
$$X^n+5X^{n-1}+3$$
is irreducible.
8. Let p be a prime number. Show that
$$X^{p-1}+X^{p-2}+\cdots+X+1$$
is irreducible.
9. Let p be a prime number, and a an integer not divisible by p. Show that the polynomial
$$X^p-X+a$$
is irreducible.
10. Show that the polynomial $(X^2+X)^{2^n}+1$ is irreducible.

Challenge: Show that if p is a prime number, then
$$\frac{(X+1)^{2p}-X^{2p}-1}{X}$$
is an irreducible polynomial.