Tiling and Coloring

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Warm-up Problem (from BAMO 2006)**:**

All the chairs in a classroom are arranged in a square *n*x*n* array. Every chair is occupied by a student. The teacher decides to rearrange the students according to the following two rules:

- Every student must move to a new chair.
- A student can only move to an adjacent chair in the same row or to an adjacent chair in the same column. In other words, each student can move only one chair horizontally or vertically. (Note that the rules above allow two students in adjacent chairs to exchange places).

For which values of *n* is this possible?

Problem 1: Remove two diagonally opposite corners of an 8x8 board.

- Can the board be tiled with dominoes (2x1's)? Why or why not?

Extension Problems:

Remove one white square and one black square from an 8x8 checkerboard.

- Is it possible to tile the remaining squares with dominoes?

Now remove 2 white squares and 2 black squares.

- Is it possible to tile this board with dominoes?

Problem 2: It is *not* possible to tile an 8x8 board *perfectly* with trominoes (3x1's). Where are the possible locations of the single "hole"? Prove that these are the *only* locations for a single hole with a coloring argument.

Problem 3: Is it possible to tile a 10x10 board with 4x1 tiles? Prove why or why not with a coloring argument.

Extension Problem:

Consider the unconventional board below. The square marked with an X is a hole in the board, so it cannot be tiled.

- Is it possible to tile the board (excluding the hole) with dominoes? Why or why not?