Counting Infinity

December 2, 2009
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Which set of numbers is bigger, A or B?

	A	B
1)	positive odd integers	positive even integers
2)	positive integers divisible by 3	positive integers divisible by 7
3)	positive integers	positive even integers
4)	positive integers and 0	positive integers
5)	positive integers	all integers
6)	positive integers	positive fractions

Recommended reading: Infinity and the Mind, by Rudy Rucker

Problems

1. On Aleph Drive there are $\aleph_0$ lots, numbered $1, 2, 3, 4, 5, \ldots$. On each lot is a Hilbert Grand Hotel with $\aleph_0$ rooms. How many rooms are there in total? $\aleph_0$? c? or something else? Prove it! ($\aleph_0$ is the cardinality of the set of integers, and c is the cardinality of the continuum)
2. Find a bijection between the set of positive integers that are divisible by 7 and the set of positive integers that end in the digit 1.
3. Find a bijection between the set of real numbers between 0 and 1 and the set of real numbers between 0 and 2.
4. Find a bijection between the set of real numbers between 0 and 1 and the set of real numbers between -1 and 1.
5. Find a bijection between the set of real numbers between 0 and 1 and the set of all positive real numbers.
6. Find a bijection between the set of all real numbers and the set of positive real numbers.
7. Find a bijection between the set of fractions that are greater than 0 and less than 1 and the set of fractions that are greater than 1.
8. Let $\mathbb{R}$ be the set of real numbers. Let $\mathbb{R}^2$ be the set of ordered pairs (x, y) where x is a real number and y is a real number. Does $\mathbb{R}^2$ have the same cardinality as $\mathbb{R}$? Hint: think of Hilbert's Hotel and interweaving the bus license numbers and the seat numbers.

9. What is the cardinality of the set of infinite sequences of positive integers? For example, $1, 2, 15, 97, 46, 2, 3, 15, 9782, 141117, 4, 2, 1, \ldots$ would be one infinite sequence, and $5, 6, 8, 19, 37, 4999, 2, 5, 8598, \ldots$ would be another.