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POLYHEDRA (3-POLYTOPES)

Three-dimensional polytopes are sometimes called *polyhedra*.

v	e	f
8		6
6		8
5	8	
	8	5
6		6
7	15	
12	30	
24	36	

(1) The table above is a partial list of the number of vertices, edges, and faces (v, e, f) of some three-dimensional polytopes. Fill in the missing entries.

(2) For each row of the table, can you find some polytope that has the corresponding numbers v, e, f? For example, the first row might be a cube.

(3) Suppose a polyhedron has 60 faces, all of them triangles. What are v, e, and f?

(4) Suppose that every face of a polyhedron is either a hexagon or a pentagon. Then how many pentagonal faces are there? Is there enough information to tell how many hexagonal faces there are?

POLYCHORA (4-POLYTOPES)

Four-dimensional polytopes are sometimes called *polychora*.

v	e	f	h
5	10	10	5
16	32	24	8
8	24	32	16
24	96	96	24
600	1200	720	120
120	720	1200	600

(1) The table above is a list of the number of vertices, edges, faces, and hyperfaces (v, e, f, h) of some four -dimensional polytopes. These particular polytopes actually have names: the 4-simplex, hypercube (tesseract), cross-polytope (octaplex), 24-cell, 120-cell, and 600-cell, respectively. What patterns do you notice among these numbers?

(2) Try to find (v, e, f, h) for a pyramid over a 3-dimensional cube, icosahedron, and dodecahedron.

(3) Find (v, e, f, h) for a bipyramid over an octahedron.

(4) What about for a prism over a cube? Think about why this is the same thing as a square "times" a square.

(5) What are v, e, f, h for a pentagon "times" a pentagon? (Can you find formulas for v, e, f, h in the product of an m-gon and an n-gon?)