It's raining Cats $\cdots$

We will discuss the TOP-SPIN puzzle. An on-line version is available at http://www.passionforpuzzles.com/virtualcube/topspin/index.php

1. Try to solve the TOP-SPIN puzzle.
2. Multiply the following permutations.
   (a) $\begin{pmatrix}1&2&3&4\\3&1&4&2\end{pmatrix}\cdot\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix}$
   (b) $\begin{pmatrix}1&2&3&4&5&6&7&8\\3&5&4&2&6&1&7&8\end{pmatrix}\cdot\begin{pmatrix}1&2&3&4&5&6&7&8\\2&4&6&8&1&3&5&7\end{pmatrix}$
3. What do several 'cat like' moves do to the puzzle?
4. Write the following permutations as products of disjoint cycles.
   (a) $\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}\cdot\begin{pmatrix}1&2&3&4\\4&3&1&2\end{pmatrix}$
   (b) $\begin{pmatrix}1&2&3&4&5&6&7&8\\2&7&4&3&6&1&5&8\end{pmatrix}$
   (c) $(14)(24)(34)$
   (d) $(123456)^5$
   (e) $(123456)^3$
   (f) $((12345)(678))^2$
   (g) $((12345)(678))^{21}$
5. Find some sequences of moves represented by small cycles by repeating some 'cat like' moves.
6. Solve the 19 number puzzle.
7. Let f be a permutation and define $\text{sign}(f) = \frac{(f(1)-f(2))(f(1)-f(3))(f(2)-f(3))\cdots}{(1-2)(1-3)(2-3)\cdots}$.
   (a) How does $\text{sign}(f)$ relate to the number of pairs of numbers that f switches?
   (b) How does $\text{sign}(fg)$ relate to $\text{sign}(f)$ and $\text{sign}(g)$?
   (c) In the 20 number puzzle what is the sign of the move that moves all numbers? What about the move that moves just 4?
   (d) Can you find positions in the 20 number puzzle that can not be fixed?