

# Counting and Symmetry

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This talk owes much to Tom Davis, who wrote a great talk on Polya's counting theory at

<http://www.geometer.org/mathcircles/index.html>

If you want to learn more after this talk, the `polya.pdf` paper there is a great choice! Lots of my material here is shamelessly stolen (with permission) from that source.

**Problem 1.** How many ways are there to put  $n$  different beads in order?

**Problem 2.** What if the beads are in a circle, so rotations don't matter?

**Problem 3.** What if the beads are on a bracelet, which can be rotated or flipped over?

**Problem 4.** Redo problem 1 through 3 with  $r$  red beads and  $g$  green beads. Hm, maybe that's too hard. Start with one red and one green, and work your way up. Can we solve it with 14 red and 6 green beads? Would it be much easier with 5 green instead of 6?

See how much harder it gets once there's symmetry?

**Problem 5.** How many ways are there to paint a six-sided die if you have one color of paint? Two colors? Three colors?  $n$  colors?

**Problem 6.** Now try the much harder problem: painting a cube with six identical faces.

**Problem 7.** How many different ways are there to put six identical beads around a bracelet? What if there are 5 of one color and 1 of another? Or 4, 2? Or 4, 1, 1? How about 3, 2, 1? How about all the ways using at most 3 different colors? (Analogy to chemistry)

**Problem 8.** My old favorite tie-dye shirt problem: how many ways are there to make a shirt with  $k$  vertical stripes if you have  $n$  colors to choose from? (First, what shirts are "the same"? Is this a hard problem even with no symmetry at all?)

**Problem 9.** Let's back up to some easier problems: how many ways are there to color a triangle with  $k$  colors? (Four possible answers: with no symmetry, rotation only, reflection only, or rotation and reflection.)

**Problem 10.** Same question, with a square.

**Problem 11.** How about an  $n$ -gon?

**Problem 12.** How many graphs are there on  $n$  vertices? (Start with small numbers, and compare doing it by hand with the fancier methods!)