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Paul Zeitz, zeitz@usfca.edu

Eight Mathematical Games

For each of these except for the last game (Cat and Mouse), two players alternate turns. The winner is the last player who makes a legal move. See if you can find a winning strategy for one of the players. Try to prove that your strategy works. And, always, try to generalize!

1 *Breaking the Bar.* Start with a rectangular chocolate bar which is 6×8 squares in size. A legal move is breaking a piece of chocolate along a single straight line bounded by the squares. For example, you can turn the original bar into a 6×2 piece and a 6×6 piece, and this latter piece can be turned into a 1×6 piece and a 5×6 piece. What about the general case (the starting bar is $m \times n$)?

2 *Takeaway.* A set of 16 pennies is placed on a table. Two players take turns removing pennies. At each turn, a player must remove between 1 and 4 pennies (inclusive).

3 *Putdown.* Each player takes turns placing a penny on the surface of a rectangular table. No penny can touch a penny that is already on the table. The table starts out completely bare.

4 *Color the Grids.* You start with an $n \times m$ grid of graph paper. Players take turns coloring red one previously uncolored unit edge of the grid (including the boundary). A move is legal as long as no closed path has been created.

5 *Brussels Sprouts.* Start by putting a few crosses on a piece of paper. On each move, a player can connect the two endpoints of a cross together, with a single line (which can be curved). Then a new cross is drawn on this connection line. You cannot ever draw a line that intersects another already-drawn line. Here are the first few moves of a sample game which started with three crosses.

start

Move 1

Move 2

Move 3
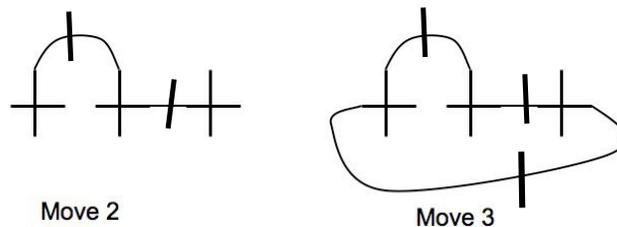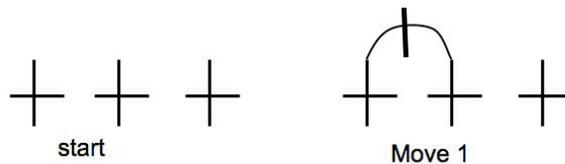
6 *Nim*. Start with several piles of beans. A legal move consists of removing one or more beans from a pile.

   (a) Verify that this game is *very* easy to play if you start with just one pile, for example, of 17 beans.

   (b) Likewise, if the game starts with two piles, the game is quite easy to analyze. Do it!

   (c) But what if we start with three or more piles? For example, how do we play the game if it starts with three piles of 17, 11, and 8 beans, respectively? What about four piles? More?

7 *Puppies and Kittens*. We start with a pile of 7 kittens and 10 puppies. Two players take turns; a legal move is removing any number of puppies or any number of kittens or an equal number of both puppies and kittens.

8 *Cat and Mouse*. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.
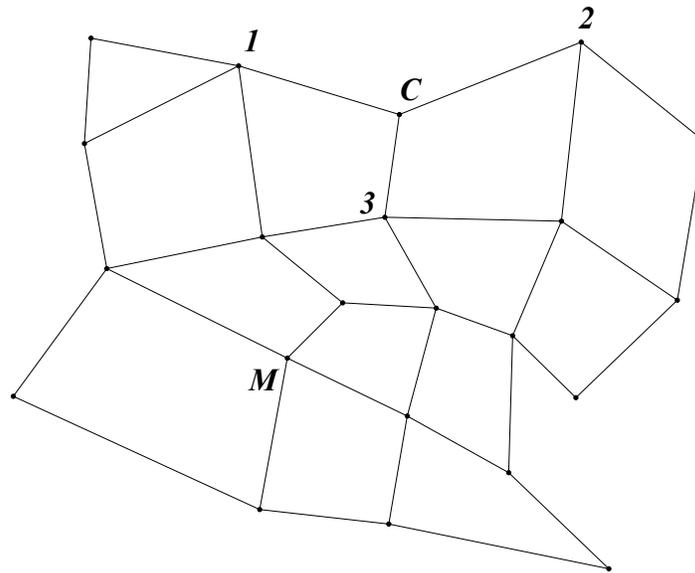
Initially, the cat is at the point labeled *C*; the mouse is at *M*. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?