

The Shape of Space: Part 1

Marin Math Circle

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linda@marinmathcircle.org

Most of this material is from *The Shape of Space* by Jeff Weeks. The word searches were taken from Torus Games at <http://geometrygames.org/TorusGames/index.html> . Download Torus Games for more word searches and many other games.

1 Dimension, Finite and Infinite, Boundary

Definitions:

- **1-dimensional:** Only one number is required to specify a location; has length but no area.
- **2-dimensional:** Two numbers are required to specify a location; has area but no volume.
- **3-dimensional:** Three numbers are required to specify a location; has volume.
- **n-dimensional** n numbers are required to specify a location.
- **Boundary:** An edge of space. A traveler who reaches a boundary can go no further.
- **Finite:** Has a limited, measurable length/area/volume/
- **Infinite:** Has unlimited length/area/volume.

1. What dimension are these?

- Your desktop
- A straight line
- The circumference of a circle
- The inside of a circle
- The surface of your skin
- The air inside this room
- The surface of a doughnut
- The inside of a doughnut

2. How many numbers are needed to specify exact time on a given day? Is time 1-dimensional, 2-dimensional, or 3-dimensional?
3. * What is the dimension of our universe? Does it have boundary? Is it finite or infinite?

2 Topology

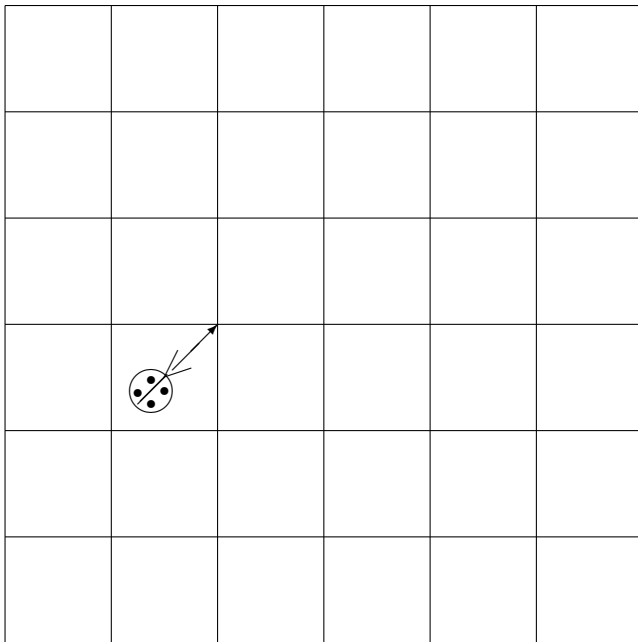
Informal Definitions:

The properties of a space that stay the same when you bend, stretch or twist it are called the *topology* of the space. Two spaces are considered the same topologically if you can deform one into the other without tearing, cutting, pinching, gluing, or other violent actions.

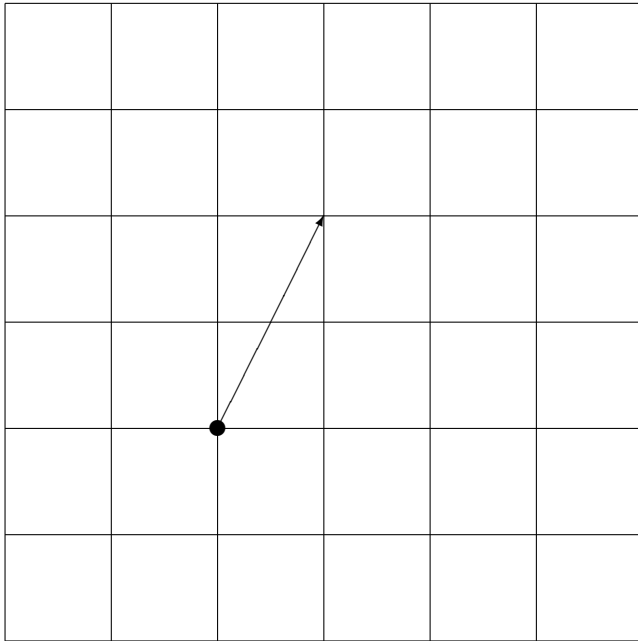
The properties of a space that change when you bend, stretch, or twist are the *geometry* of the space. For example, distances, angles, and curvature are parts of geometry but not topology.

3 Ladybug Trails

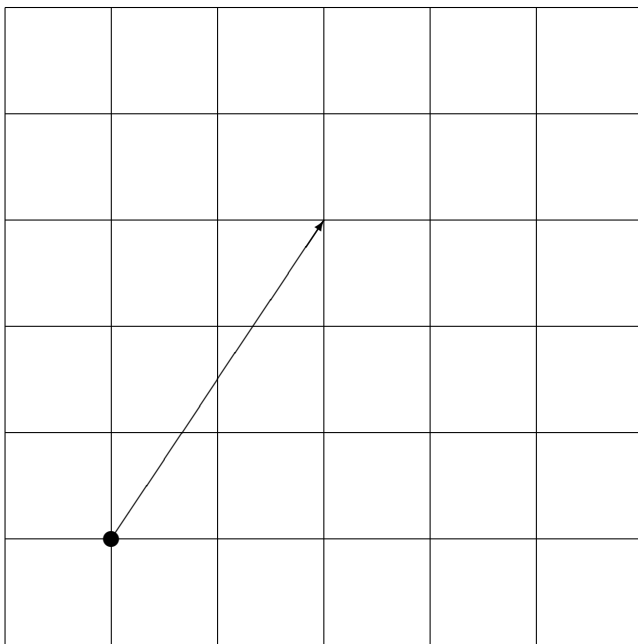
4. A ladybug on a torus walks in a straight line until she returns to her starting point. She walks 1 unit northward for every 1 unit eastward. Draw her path. What is the length of her path?



5. Next the ladybug walks 2 units northward for every 1 unit eastward as shown. Draw her path. What is the length of her path?



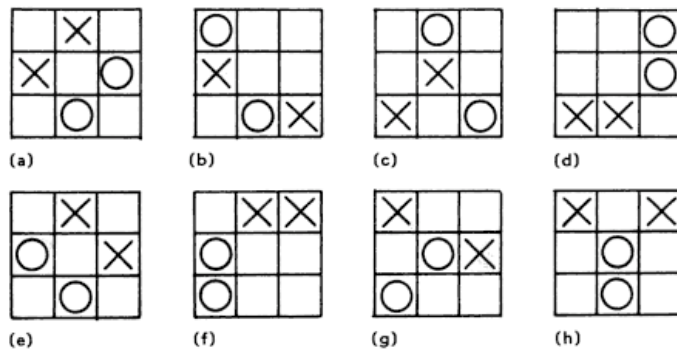
6. Next the ladybug walks 3 units northward for every 2 units eastward as shown. Draw her path. What is the length of her path?



7. If the ladybug walks n units northward for every e units eastward, where n and e are integers, how long will her path be? What if n and e are not integers? Will she always return to her starting place eventually, or is it possible for her to walk forever in a straight line and never get back home?

4 Tic Tac Toe on a Torus and Other Games

8. A Cats Game in Tic Tac Toe is a game where neither side wins, even though the board is filled up with Xs and Os. Is it possible to have a Cats Game in Tic Tac Toe on the torus?
9. How many essentially different opening moves does the first player have in Tic Tac Toe on the torus?
10. If the first player in torus tic-tac-toe takes the upper left corner, how many nonequivalent moves does the second player have to choose from?
11. Is there a winning strategy for the first player in Tic Tac Toe on the torus? That is, is it possible for the first player to win no matter what the second player does? Does it change your answer if the first player is required to start on the center square?
12. Which of the following positions are equivalent in torus tic-tac-toe?



13. Try this word search on the torus. Remember that the top and the bottom are glued together and the left and right sides are glued together.

bat	pig
camel	possum
cat	puma
jaguar	rat
llama	seal
ox	shrew
panda	tapir

