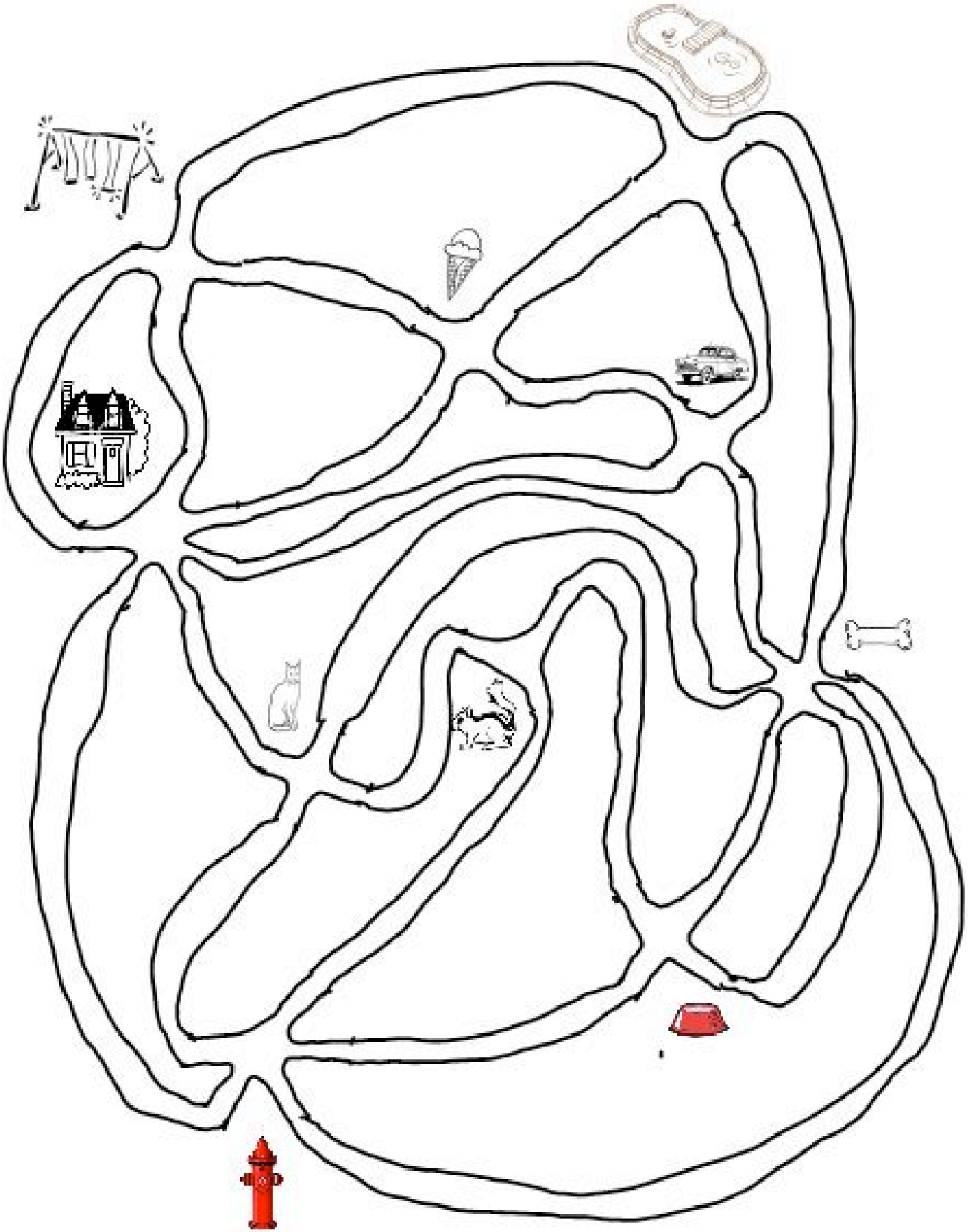
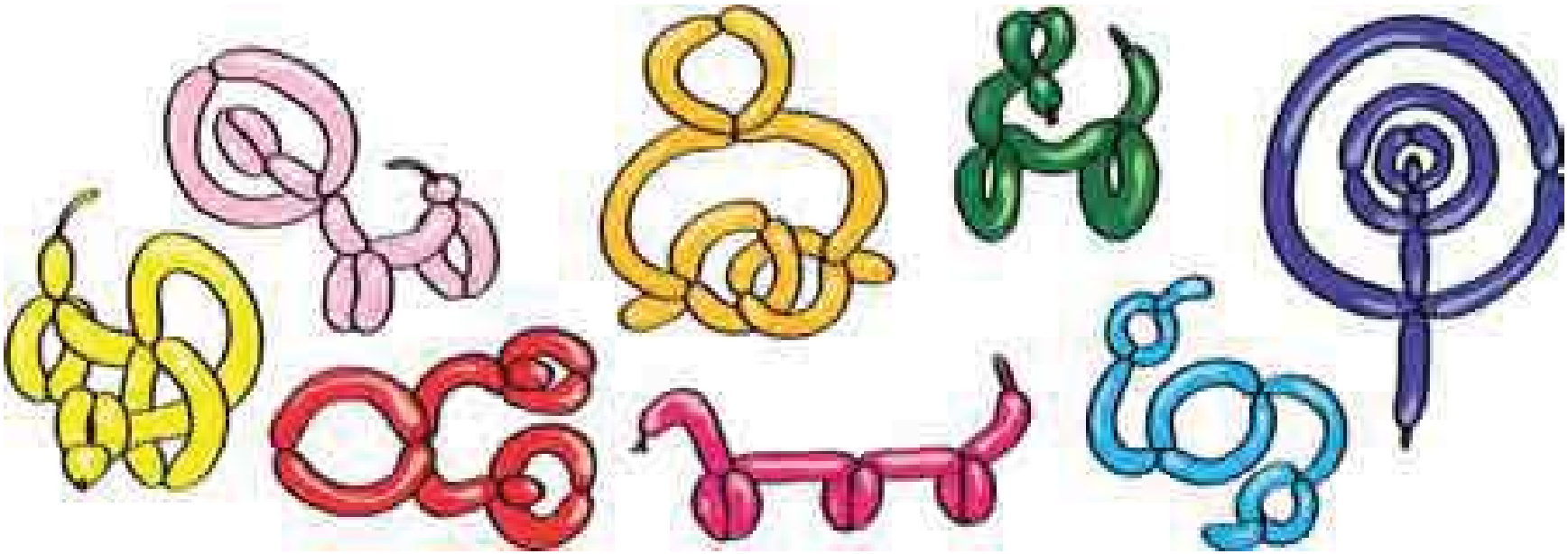


Help puppy explore by finding a walk that starts at one of the intersections and proceeds over each segement of sidewalk exactly once.

Which of these are Dogs?



Julia Robinson Mathematics Festival

Balloon Numbers

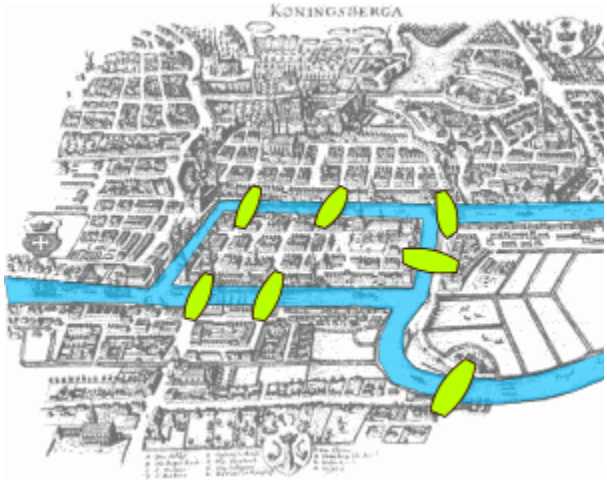
This activity is based on

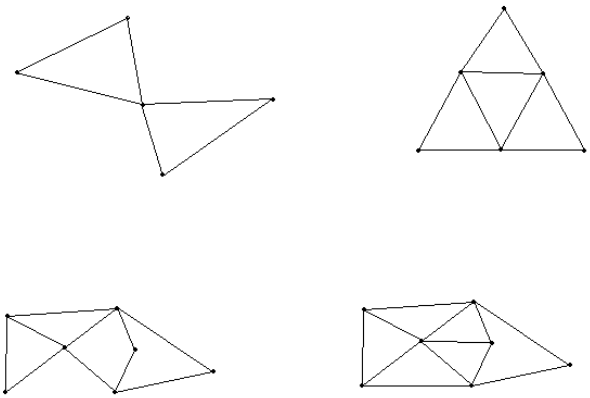
"Computational Balloon Twisting: The Theory of Balloon Polyhedra"
by Erik D. Demaine, Martin L. Demaine, and Vi Hart
in Proceedings of the 20th Canadian Conference on Computational Geometry (CCCG 2008), Montréal, Québec, Canada, August 2008

See http://vihart.com/balloons/ as well.

- Try to make a cube using balloons. How many balloon ends do you need to make a corner, so how many balloons do you need? Draw your design first.

A famous problem in mathematics is the Seven Bridges of Königsberg problem. Königsberg was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. (See the map below.)



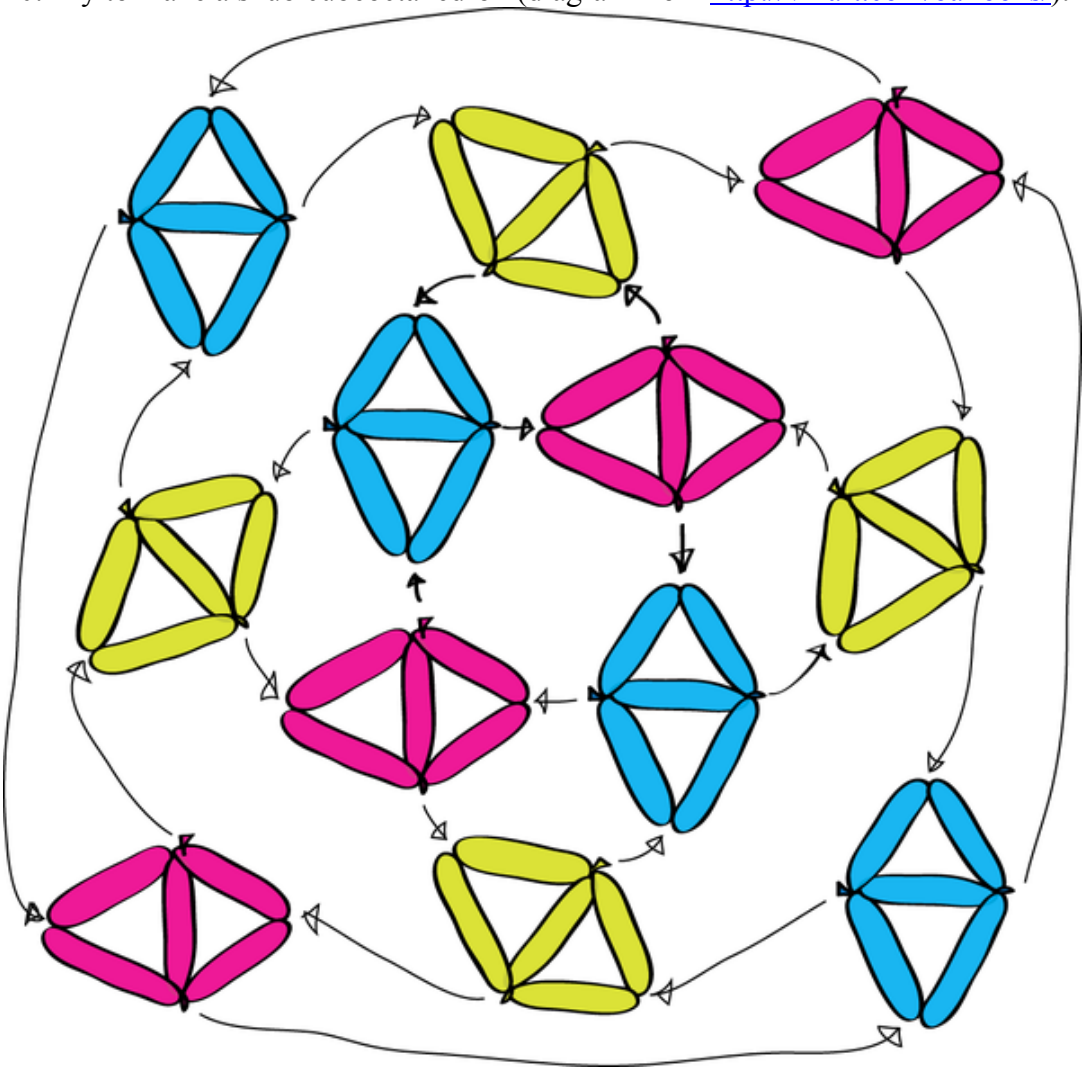


http://en.wikipedia.org/wiki/File:Konigsberg_bridges.png

The problem was to find a walk through the city that would cross each bridge once and only once.

1. Try to find such a walk. Try to trace each edge of the above graphs exactly once without lifting your pen or tracing an edge twice. Can you end up where you start?
2. How many ends does a balloon have?

3. For each of the graphs you traced, count the number of ends of edges at each vertex (this number is called the degree of the vertex). Add up the degrees for all the vertices in each graph. How many vertices have odd degree?
4. What can you say about the parity of the number of ends of edges in a graph? (In other words, is this number even 2, 4, 6…, or can it be odd 1, 3, 5…?) Why?
5. What can you say about the parity of the number of vertices in a graph that meet an odd number of ends of edges?
6. Can you twist a balloon so that the result has no ends?
7. Can you make an Octahedron (eight triangles glued together along edges so that four triangles meet at every vertex) out of one balloon?
8. If a graph has 6 vertices that each meet an odd number of ends of edges, what can you say about the number of balloons required to make such a graph.
9. What is the smallest number of balloons required to make a tetrahedron? a cube? an icosahedron? a dodecahedron?
10. Try to make a snub cubeoctahedron (diagram from http://vihart.com/balloons/):



11. How would the balloon number change if you were allowed to delete several balloon edges after making your creation?