MATH AUCTION

October 19, 2011

Rules:

1) We divide into teams and work for a fixed amount of time to solve the problems below.

2) Each team is given $500 to start.

3) The best solution to a problem is worth $200.

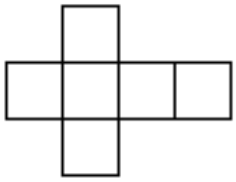
4) The problems are put up for auction in the order given. The team with the highest bid is allowed to present its solution.

5) The problem is then put up for bid again (and again), but each time the solution must be better than the previous solution.

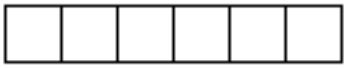
6) When no other team wants to buy the problem, the team with the best solution collects the value of the problem. Every team that "bought" the problem pays for its bid, even if it did not have the winning solution.

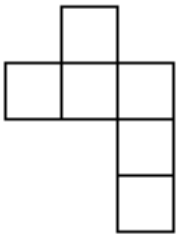
7) If a team can show that it has found the best solution (by showing that no better solution is possible), then that team gets an additional $50 prize money for the problem.

1) The following diagram, called a "net," folds to make a cube:



whereas the following two diagrams do not:



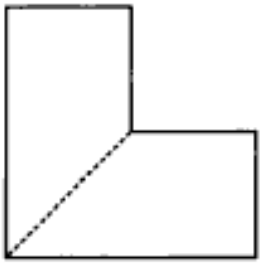


How many essentially different nets can you find that make a cube? (Each net consists of six squares connected by whole edges. Two nets that are the same after rotating and / or reflecting are essentially the same, not essentially different.)

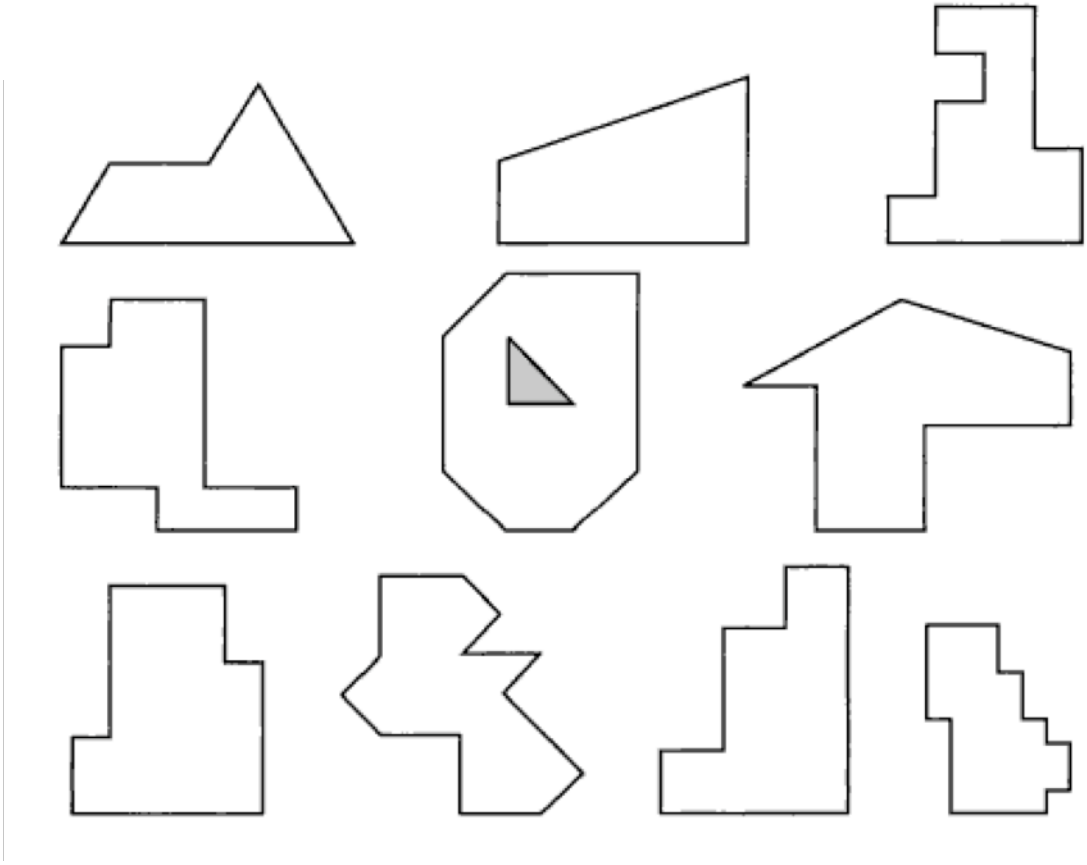
2) Ali the trader has a heavy gold bracelet with 11 links in it. During one of his trips, he decides to take a break from work and stay at a local inn for 11 days. Since Ali has no money, the innkeeper asks him for one gold link per day as payment. The trader does not want to pay more that he owns, while the innkeeper wants to get his payment daily. However, the innkeeper is willing to trade the links to make even. For example, if Ali pays with the a single link on day one, he can give a chain with two links on day two and get the single link back. Therefore, the trader will have to cut his bracelet into several pieces by breaking a few of its links. What is the minimum number of links Ali needs to cut?

3) How many different subsets of the set {1, 2, 3, 4, 5, 6, 7} contain no two successive numbers? (A subset is any collection of these numbers. For example, {2, 6} is a subset with no two successive numbers. The set {2, 5, 6} is a subset, but it contains two successive numbers, 5 and 6. The empty set {} containing none of the numbers counts as a subset with no successive numbers.)

4) This L shape can be divided into two congruent halves. (Mirror images are considered congruent.)



Divide as many of these shapes as you can into two congruent halves.



5) A military base has a number of identical hoverplanes. Each hoverplane can carry enough fuel to fly exactly halfway around the planet. Hoverplanes do not use any fuel while hovering stationary in the air, and hoverplanes can transfer any amount of fuel between each other while in the air. What is the minimum number of planes are that are needed so that one plane is able to get all the way around the planet and all assisting planes return safely to base.

Thank you to the Boston Math Circle for problem 1, Anna Buraga for problem 2, the National Association of Math Circles for problem 3, *Solve This* by James Tanton for problem 4, and the Bay School for problem 5.