

Combinatorics - II

Marin Math Circle

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Some of these problems are from *Mathematical Circles (Russian Experience)*

1 Warm-up Problems

1. Susanna owns 6 different shirts, 5 different pairs of pants, and 3 pairs of shoes. How many different outfits can she wear? (An outfit consists of a shirt, a pair of pants, and a pair of shoes, and she refuses to borrow clothes or wear mismatched shoes.)
2. A pet store has 4 different puppies, 7 different kittens, and 3 different Komodo dragons for sale. How many ways can you pick 2 animals of different species to bring home?
3. How many different ways can you sew a three-colored flag with 3 horizontal stripes of equal height if you have six colors of fabric to work with? We can distinguish the top of the flag from the bottom.

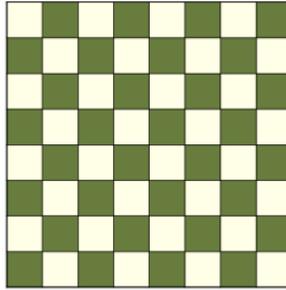


4. If you roll a six sided die three times, how many different possible outcomes are there? (For example, 1, 6, 5 is one outcome, and 6, 6, 6 is another.)
5. You have five starburst candies, each a different flavor: Cherry, Lemon, Lime, Orange, Strawberry . How many different flavor combinations can you make by putting one or more candies in your mouth? For example, one combination is Cherry-Lemon-Orange.



6. How many different ways can you pick a president and vice president from a club of 10 kids?
7. How many different ways can you pick a pair of co-presidents from a club of 10 kids?

8. I have three kids who need to take turns getting flu shots. How many different orders are possible?
9. How many different license plates can you make with 3 letters followed by 3 digits? Would there be enough to use this system for the cars in California?
10. How many ways are there to arrange the letters in the word CANOE? How many of these arrangements spell an English word?
11. How many ways are there to put one white rook and one black rook on a chessboard so that they do not attack each other?



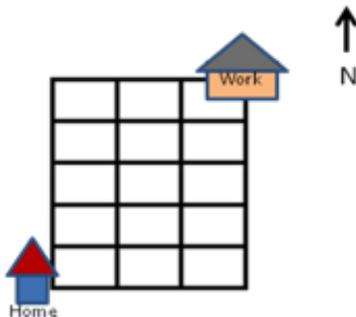
12. How many ways are there to put one white king and one black king on a chessboard so that they do not attack each other?

2 New Tricks - Divide by the Overcount

- How many ways are there to arrange the letters in the word HOGWARTS? The letters in the word VOLDEMORT? The letters in the word ALOHOMORA? The letters in the word AVADAKEDAVRA?
- You have 2 identical Reece's pieces, 4 identical mini bags of M&M's, and 1 sour candy left from Halloween. You are going to eat one candy per day until the candy is gone. How many different ways can you do this?
- How many ways are there to choose a team of three students out of a class of 30?
- How many ways can you make a pizza with 3 different toppings if there are 8 toppings to choose from?



- In how many ways can you choose a debate team of 6 students from Ms. Jewls' class of 11 girls and 17 boys? What if the team has to consist of 3 girls and 3 boys?
- Ten points are marked on a plane so that no three of them are on the same straight line. How many triangles are there with vertices at these points?
- How many different routes are there from home to work, only traveling north and east on streets?



- Ms. Jewls' class is going to play capture the flag at recess. How many ways are there to divide the 28 students into 2 teams of 14? What if Maricia and Deedee can't be on the same team?
- What is the largest number of triangles you can make by drawing 7 lines in the plane? The triangles may overlap or contain each other.

Permutations:

Permutations: The number of ways to lay out n different objects in a row is

$$n! = n \cdot (n - 1) \cdot (n - 2) \dots 3 \cdot 2 \cdot 1$$

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Combinations:

The number of ways to choose k objects out of a collection of n objects is

$$\binom{n}{k} = \frac{n \cdot (n - 1) \cdot (n - 2) \dots (n - k + 1)}{k!} = \frac{n!}{k!(n - k)!}$$

3 Extra Problems

22. You roll a 6-sided die 3 times. Among all possible outcomes, how many have at least one occurrence of the number 6?
23. How many five digit numbers have an even sum of their digits?
24. You flip a coin 10 times. Of all the possible outcomes, how many have exactly 5 heads in a row? For example, we would not count HHHHHHHTTT (too many consecutive heads), but we would count TTTHHHHHTT and HHTTHHHHHT.
25. How many even three digit numbers have no repeating digits?
26. There are 6 boys and 6 girls in a dance class. In how many ways can they partner off into boy-girl couples?
27. All 11 girls in Ms. Jewls' class are going to the circus, and will be sitting in a row of 11 seats. How many ways are there for them to seat themselves if Mauricia and Deedee refuse to sit next to each other?
28. How many ways are their to seat 9 girls and 10 boys in a row if all boys must sit together and all girls must sit together? What if boys and girls must alternate?
29. There are 4 boyfriend-girlfriend couples in a drama club. How many ways are there to choose a stage crew of 3 members so that the stage crew doesn't contain a boyfriend-girlfriend couple?
30. How many ways are there to rearrange the letters in the word "FLAMINGO" so that the vowels will be in alphabetical order and so will the consonants? For example, FAGILMON (A - I - O, F - G - L - M - N).
31. How many ways are there to distribute 10 doggie biscuits among 7 dogs? The biscuits are indistinguishable, but the dogs are distinguishable.