Marin Math Circle Combinatorics Dash

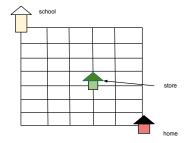
The Dash Instructions. 3 points will be awarded for the first correct answer to each question; 2 points will be awarded for the second correct answer to each question; 1 point will be awarded for the third correct answer to each question. Write your team name, the problem number, and your answer (in simplest form) on your slip of paper before leaving your seat. You may not work on questions while waiting to show your answer to the judges. Thus, you should bring only your solution slip with you when you dash to the front. You may bring only one solution slip at a time. Good luck, and have fun!

- 1. Connor will be taking 6 classes next year: English, Drama, Teacher's Aid, Physics, Statistics, and Government. The schedule at his high school has 7 periods, so he will have one period free. How many different possible schedules could he have?
- 2. You toss a coin 5 times. How many different sequences of heads and tails can you get?
- 3. Each of two novice collectors has 20 different stamps and 10 different postcards. We call an exchange fair if they exchange a stamp for a stamp or a postcard for a postcard. How many ways are there to carry out one fair exchange between these two collectors?
- 4. How many ways are there to choose a vowel and a consonant from the word ARTICHOKE?
- 5. Each box of a 2 x 2 table can be colored black or white. How many different colorings of the table are there?
- 6. Eight points are marked on a plane so that no three of them are on the same straight line. How many triangles are there with vertices at these points?
- 7. You are organizing a talent show with 8 different acts. In how many different orders can the acts be arranged, if the fire-eating unicycle rider has to be scheduled in the first half because she has to leave early?
- 8. How many ways are there to divide 10 students into 2 teams of 5?
- 9. How many different four-digit numbers can be written using the digits 1, 2, 3, 4, if each digit can be used only once?
- 10. How many different four-digit **even** numbers can be written using the digits 1, 2, 3, 4, if each digit can be used only once?
- 11. How many different four-digit **even** numbers can be written using the digits 1, 2, 3, 4 if each digit can be used any number of times?
- 12. How many ways can you play 5 songs in a row on your iPod, if you have 15 songs total, and you don't want to repeat any songs?
- 13. How many different types of milkshake can you make if a milkshake must contain exactly 3 different flavors of ice cream and you have 10 flavors total in your kitchen?
- 14. The rules of a soccer tournament say that each team must play each other team exactly once. How many games will be played if there are 18 teams?
- 15. Five points are marked on a straight line and 7 points are marked on another straight line, parallel to the first one. How many triangles can be made with vertices at these points?

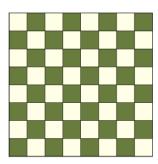
Marin Math Circle Combinatorics Dash

16. How many ways are there to pick 1 or more math books to take on vacation if you have a shelf of 10 books to choose from?

- 17. There are 6 boys and 6 girls in a dance class. In how many ways can they partner off into boy-girl couples?
- 18. How many "words" can be written using exactly 5 letters A and 4 letters B? (Here, a "word" is just any arrangement of letters, like ABBABAAAB, for example.)
- 19. How many different routes are there from school to home, if you need to stop by the store on the way, and you can only traveling south and east on streets?



20. How many ways are there to put one white rook and one black rook on a chessboard so that they do not attack each other?



- 21. How many ways are there to put one white king and one black king on a chessboard so that they do not attack each other?
- 22. You flip a coin 6 times. Of all the possible outcomes, how many have more heads than tails?
- 23. You roll a 6-sided die 3 times. Among all possible outcomes, how many have at least one occurrence of the number 6?
- 24. There are three rooms in a dormitory: one single, one double, and one for four students. How many ways are there to house seven students in these three rooms?
- 25. How many necklaces can be made using 5 identical red beads and 2 identical blue beads?
- 26. One girl owns 8 DVDs and her sister owns 6. How many ways are there for the first girl to trade 3 of her DVDs for 3 of her sister's DVD's?

Marin Math Circle Combinatorics Dash

27. How many five digit numbers have an even sum of their digits? (Note: a number like 09763 with 5 digits that starts with the digit 0 is considered a 4 digit number, not a 5 digit number.)

- 28. How many six digit numbers have at least one even digit? (Note: a number like 064862 with 6 digits that starts with the digit 0 is considered a 5 digit number, not a 6 digit number.)
- 29. What is the largest number of triangles you can make by drawing 5 lines in the plane? The triangles may overlap or contain each other.
- 30. You flip a coin 10 times. Of all the possible outcomes, how many have exactly 5 heads in a row? For example, we would not count HHHHHHHTTT (too many consecutive heads), but we would count TTTHHHHHHTT and HHTTHHHHHHTT.
- 31. How many even three digit numbers have no repeating digits? (Note: a number like 097 with 3 digits that starts with the digit 0 is considered a 2 digit number, not a 3 digit number.)
- 32. All 11 girls in Ms. Jewls' class are going to the circus, and will be sitting in a row of 11 seats. How many ways are there for them to seat themselves if Mauricia and Deedee refuse to sit next to each other?
- 33. There are 4 boyfriend-girlfriend couples in a drama club. How many ways are there to choose a stage crew of 3 members so that the stage crew doesn't contain a boyfriend-girlfriend couple?
- 34. How many ways are there to rearrange the letters in the word "FLAMINGO" so that the vowels will be in alphabetical order and so will the consonants? For example, FAGILMON (A I O, F G L M N).
- 35. How many ways are there to distribute 10 doggie biscuits among 7 dogs? The biscuits are indistinguishable, but the dogs are distinguishable. Hint: line up all doggie biscuits in a row, and put dividers in between them to mark which ones will go to which dog. For example, if the dividers and biscuits are arranged as drawn below, the first dog will get 1 biscuit, the second dog 3 biscuits, the third dog 1 biscuit, the fourth dog 2 biscuits, the fifth dog 0 biscuits, the sixth 3 biscuits, and the seventh 0. How many ways are there to do this?



- 36. How many ways are there to distribute 3 one dollar bills and 10 quarters among 4 different children?
- 37. How many ways are there to represent the number 10 as a sum of 5 non-negative integers? The order of the numbers matters here, so, for example, 1 + 4 + 4 + 0 + 1 is considered different from 0 + 1 + 1 + 4 + 4.
- 38. *How many ways are there to represent the number 10 as a sum of 5 positive integers? The order of the numbers matters here, so, for example, 1 + 4 + 3 + 1 + 1 is considered different from 1 + 1 + 1 + 3 + 4.
- 39. *How many ways are there to rearrange the letters in the word "FLAMINGO" so that the vowels will be in alphabetical order and so will the consonants? For example, FAGILMON (A I O, F G L M N).
- 40. *How many ways are there to write 10000 as a product of 3 numbers, if the order of the factors matters? For example, 500*4*5 is considered different from 4*5*500.

Many of these problems are from Mathematical Circles (Russian Experience) and from A Decade of the Berkeley Math Circle -Volume 1