

# Trapezoidal Numbers

Marin Math Circle, March 14, 2012  
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## Compute

1. What is the sum  $3 + 4 + 5$ ?
2. What is the sum  $4 + 5 + 6 + 7 + 8$ ?
3. What is the sum  $5 + 6 + \dots + 80 + 81$ ?

All of the results of these computations are called *trapezoidal* numbers, because you can draw a trapezoid that illustrates the answer to problem 1 with dots or blocks like this:



where each row has one more dot than the row before. So for instance 13 is trapezoidal because it is equal to  $6 + 7$ . A trapezoidal number has to have at least two rows.

## Patterns

4. What numbers can be written as 2-row trapezoidal numbers, like 13?
5. What numbers can be written as 3-row trapezoidal numbers, like  $3 + 4 + 5$ ?
6. What numbers can be written as 4-row trapezoidal numbers?
7. What about 5-row, 6-row, and so on? Can you explain a general rule, so that we can tell whether 192 is a 12-row trapezoidal number?
8. Can you name a large number that is not trapezoidal, no matter what number of rows you try? How do you know it can't be trapezoidal?
9. Can you name a large number that is trapezoidal in only one way? How do you know?
10. How many trapezoidal representations does 100 have? Why? How about 1000?
11. How many trapezoidal representations does 221 have? Why?
12. How can you determine how many trapezoidal representations a number has?
13. What if we allow negative numbers, like  $-2 + -1 + 0 + 1 + 2 + 3 + 4 + 5$ , in a trapezoidal representation? What if we allow "staircases" like  $3 + 7 + 11$ ?