Marin Math Circle
HOW TO GAMBLE IF YOU MUST

March 28, 2012
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Fun with Dice

1 *Thirty-six scenarios*. When two dice are rolled, there are 36 different outcomes, because $36 = 6 \times 6$. Each outcome is equally likely. This allows us to compute probabilities easily. Use this table to compute the various sums that can occur. I filled in the first few cells.

	1	2	3	4	5	6
1	2	3	4			
2		4				
3						
4						
5						
6						12

(a) Verify that the probability of rolling two dice and getting a sum of 2 is $1/36$.

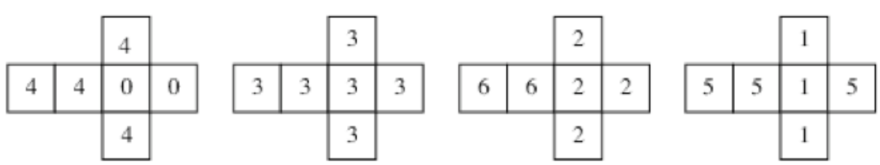
(b) What is the probability of rolling two dice and getting a sum of 5?

(c) What is the most likely sum, and what is its probability?

(d) Test these probabilities by experiment. For example, suppose you want to test for a sum of 12, which should also have a probability of $1/36$. You'd expect to see this sum happen only once every 36 or so rolls. Experiment!

2 *Non-transitive dice*. Construct the following set of four dice.

A B C D



(a) Suppose one person tosses the A die, and the other tosses the B die. The winner is the one whose die has the bigger number. What is the probability that A wins? You will need to make a 6×6 table like this. I filled in a few cells, saying who the winner is.

A vs B	0	0	4	4	4	4
3	B	B	A			
3						
3						
3						
3						
3						

(b) Work out the probabilities for the other possibilities, such as B vs. C, C vs. D, etc. Here are some tables to help you organize your work.

B vs. C	2	2	2	2	6	6
3	B	B			C	
3						
3						
3						
3						
3						

C vs. D	2	2	2	2	6	6
1	C	C				
1						
1						
5						
5	D					
5						

A vs. C	2	2	2	2	6	6
0						
0						
4						
4						
4						
4						

A vs. D	1	1	1	5	5	5
0						
0						
4						
4						
4						
4						

(c) Now that you have collected data, do you notice something strange about these four dice? Can you construct a “sucker bet” with them?

Fun with Cards

1 I deal cards onto a table, while you tell me whether the card should be face up or face down (I do what you say). You also tell me when to stop dealing the cards (it doesn’t have to be the whole deck). Then you put a blindfold on me. I bet you that my fingers are so sensitive that I can detect the difference between a face up card and a face down card by touch. I bet that I can divide the cards on the table into two piles, each of which has exactly the same number of face cards. Do you want to take this bet?

2 A deck of cards is randomly cut into three piles. I bet that at least one of the cards on the top of a pile is a “face card;” i.e., a Jack, Queen, or King? Do you want to take this bet?

3 We each have a shuffled deck of cards and we deal our cards one at a time and compare. I bet you that we will have at least one match (for example, our 7th cards are the same). Do you want to take this bet?

4 *Kruskal Count*. This one is hard to describe. I will demonstrate this amazing trick for you. Your job will be to explain why it works.

Miscellaneous Problems

1 *The Monty Hall Problem*. Like the Kruskal Count, this is easier to do than write about.

2 It costs a consumer \$1 to buy a Klopstockia lottery ticket. The buyer then scratches the ticket to see the prize. Compute, to the nearest penny, the expected profit that the state of Klopstockia makes per ticket sold, given the following scenarios for prizes awarded. (The state will make a profit if the expected value of the lottery ticket is *less* than \$1.)

(a)

Prize	\$1	\$10	\$1000
Probability	$\frac{1}{10}$	$\frac{1}{1{,}000}$	$\frac{1}{1{,}000{,}000}$

(b)

Prize	\$1	\$10	\$1000	a free lottery ticket
Probability	$\frac{1}{10}$	$\frac{1}{1{,}000}$	$\frac{1}{1{,}000{,}000}$	$\frac{1}{5}$

3 Two players alternately toss a penny, and the one that first tosses heads wins. What is the probability that

(a) the game never ends?

(b) the first player wins?

(c) the second player wins?

4 *What a Loser!* You arrive in Las Vegas with \$100 and decide to play roulette, making the same bet each time, until you are either bankrupt or have doubled your money. Which of the following strategies is best?

(a) Making bets of \$1 each time.

(b) Making bets of \$10 each time.

(c) Making a single bet of \$100.

5 *A Gambling "System."* Suppose you are playing a game with a 50% chance of winning each time. You can bet any amount, and if you win, you win twice your bet. If you lose, you lose your bet. In other words, if you bet B dollars, your *profit* is $\pm B$ depending on whether you win or lose. You decide that you will play, stopping as soon as you win, doubling the size of your bet each time. You are guaranteed to make a profit? Right? Use expectation to show that this won't work. What if you triple instead of double?

6 *The St. Petersburg Paradox*. Consider the following game. I will flip a fair coin until it shows up heads. We keep track of the number of flips until this happens. If it happens on the first flip, I'll pay you \$2. If it takes two flips, then I'll pay you \$4. Three flips, \$8, etc. In other words, if it takes n flips until the first head, I will pay you 2^n dollars. Pretty sweet game!

How much is this game worth *to you*? In other words, if a there was a ticket that allowed you to play the game once with me (I flip the coin until it is heads, and pay you the appropriate amount), how much would you pay for the ticket? Clearly, you'd pay at least 1 dollar. In fact, you'd almost certainly pay 2 dollars. How about 3? 4? 5? More?