Determined Games
Or How to Win a Cookie

Determined games are games where only one player can have a winning strategy. You can win every time if you know what you're doing and the game starts in a favorable position for you.

Your job is to play correctly if you're in the right spot and hope your opponent doesn't if you're not, although hope is not an operational strategy. My job is to show you the right positions and how to play correctly.

Sprouts: Sprouts is one such determined game. In Sprouts, the game starts with some number of points. Players alternate turns. The player whose turn it is draws a line between two points and puts a point on that line such that each point has at most three lines coming out of it.

In one version of Sprouts, the lines can cross each other; in the more well known version, they can't. The first person unable to move (by connecting two dots with a line and adding a dot on that line such that no dot has more than three lines coming out of it), loses the game. In all the other games we'll look at below, the first player unable to move always loses. The other one is in line for a cookie if that's what's on the table. Get your parents' permission before ever betting.

Problem 1: At most, how many moves do the two versions take if you start with n points (n = any whole number)? At least how many? What can you conclude about the first version, the one that allows lines to cross over other lines?

Nim: Nim and other Nim-like games are games where players subtract a number in a specified set of whole numbers from some total count. Again, the first player unable to move loses. For example, one Nim-like game is a game where you can subtract any power of two from the total count, starting at some specified number. Here is an example is this game:

Ex. The count starts at 39.

> Player 1 subtracts 32, the count is now 7.
> Player 2 subtracts 4,   the count is now 3.
> Player 1 subtracts 1,   the count is now 2.
> Player 2 subtracts 2,   the count is now 0.

Player 1 loses! Note: Player 2 was using a strategy which would earn him or her a cookie.

Problem 2: Experiment with games like this. How do you find for which starting numbers the first or second players win?

Problem 3: Can you find a simpler way of finding who wins for certain sets?

Problem 4: What happens if you have more than one total count and can subtract a certain numbers from each count each turn?

Hackenbush: Hackenbush is a game where you have a graph of red and blue edges and one vertex called the ground, usually represented by a line. Each turn, players red and blue take turns removing an edge of their color and then all edges and vertices not connected to the ground. You can determine who wins by the use of surreal numbers, but with finite games, you only need fractions with a denominator that is a power of two. Don't worry, I'll explain!

Problem 5: Who wins these games if blue goes first? What about red?
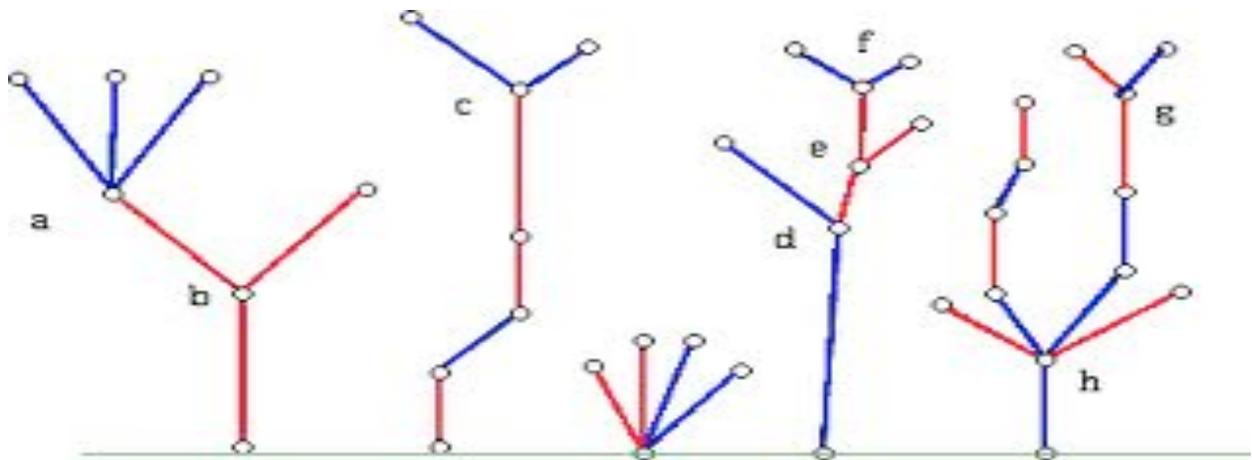
Problem 6: Can you come up with a strategy for any graph?

You can make up your own determined games, too (I'll share a couple of my own). Maybe you can bet a cookie that you'll win!

Problem 7: Make up your own games and play them!